## Solving Linear Equations in One Variable

## 1 GETTING THE IDEA

A linear equation in one variable may have one solution, infinitely many solutions, or no solutions. A solution to a linear equation is a number that, when substituted for the variable, makes the equation true. To solve an equation in one variable, use the properties of equality to isolate the variable on one side of the equation.

## Properties of Equality

For all real numbers $a, b$, and $c$ :
Addition property of equality: If $a=b$, then $a+c=b+c$.
Subtraction property of equality: If $a=b$, then $a-c=b-c$.
Multiplication property of equality: If $a=b$, then $a c=b c$.
Division property of equality: If $a=b$ and $c \neq 0$, then $\frac{a}{c}=\frac{b}{c}$.

You can check your solution to a linear equation by substituting the value of the variable back into the original equation. If a true sentence results, the solution is correct.

## Example 1

Solve for $x .2(3 x+0.5)=25$
Strategy Use the properties of equality to isolate the variable.
Step 1 Simplify the expression on the left side of the equation by applying the distributive property.

$$
\begin{aligned}
2(3 x+0.5) & =25 \\
2 \cdot 3 x+2 \cdot 0.5 & =25 \\
6 x+1 & =25
\end{aligned}
$$

Step 2 Use the subtraction property of equality. Subtract 1 from both sides of the equation.

$$
\begin{aligned}
6 x+1 & =25 \\
6 x+1-1 & =25-1 \\
6 x & =24
\end{aligned}
$$

Step 3 Use the division property of equality. Divide both sides by 6 to solve for $x$.

$$
\begin{aligned}
6 x & =24 \\
\frac{6 x}{6} & =\frac{24}{6} \\
x & =4
\end{aligned}
$$

Step 4 Check the solution by substituting 4 for $x$ into the original equation.

$$
\begin{aligned}
2(3 x+0.5) & =25 \\
2(3 \cdot 4+0.5) & \stackrel{?}{=} 25 \\
2(12+0.5) & \stackrel{?}{=} 25 \\
2(12.5) & \stackrel{?}{=} 25 \\
25 & =25
\end{aligned}
$$

Solution The equation $2(3 x+0.5)=25$ has one solution. The solution is $x=4$.

## Example 2

Solve for $k .4-2 k=-5(k+7)$

## Strategy Use the properties of equality to isolate the variable.

Step 1 Simplify the expression on the right side of the equation by applying the distributive property.

$$
\begin{aligned}
& 4-2 k=-5(k+7) \\
& 4-2 k=-5 \cdot k+(-5) \cdot 7 \\
& 4-2 k=-5 k-35
\end{aligned}
$$

Step 2 Use the addition property of equality. Add $5 k$ to both sides of the equation, and then combine like terms.

$$
\begin{aligned}
4-2 k & =-5 k-35 \\
4-2 k+5 k & =-5 k-35+5 k \\
4+3 k & =-35
\end{aligned}
$$

Step 3 Use the subtraction property of equality. Subtract 4 from both sides of the equation, and then combine like terms.

$$
\begin{aligned}
4+3 k & =-35 \\
4+3 k-4 & =-35-4 \\
3 k & =-39
\end{aligned}
$$

Step 4 Use the division property of equality. Isolate the variable by dividing both sides by 3, and then simplify.

$$
\begin{aligned}
3 k & =-39 \\
\frac{3 k}{3} & =-\frac{39}{3} \\
k & =-13
\end{aligned}
$$

Step 5 Check the solution by substituting -13 for $k$ into the original equation.

$$
\begin{aligned}
4-2 k & =-5(k+7) \\
4-2(-13) & \stackrel{?}{=}-5(-13+7) \\
4+26 & \stackrel{?}{=}-5(-6) \\
30 & =30
\end{aligned}
$$

Solution The equation $4-2 k=-5(k+7)$ has one solution. The solution is $k=-13$.

Some equations have no solutions. If you try to solve an equation that has no solution, the equation will be transformed into a number sentence that is false.

## Example 3

Solve for $y \cdot \frac{3}{4}(4 y+8)=5+3 y$

## Strategy Use the properties of equality to isolate the variable.

Step 1 Apply the distributive property.

$$
\begin{aligned}
\frac{3}{4}(4 y+8) & =5+3 y \\
3 y+6 & =5+3 y
\end{aligned}
$$

Step 2 Subtract $3 y$ from both sides of the equation.

$$
\begin{aligned}
3 y+6 & =5+3 y \\
3 y+6-3 y & =5+3 y-3 y \\
6 & =5
\end{aligned}
$$

Step 3 Interpret the resulting equation.

$$
6=5
$$

This is not a true statement, so there is no solution to the equation

$$
\frac{3}{4}(4 y+8)=5+3 y
$$

Solution The equation $\frac{3}{4}(4 y+8)=5+3 y$ has no solution.

Some equations are true no matter what the value of the variable. These equations have infinitely many solutions. If you try to solve an equation that has infinitely many solutions, the equation will be transformed into a number sentence that is true.

## Example 4

Solve for c. $10 c-6(2 c-1)=-2(c-3)$

## Strategy Use the properties of equality to isolate the variable.

Step 1 Apply the distributive property to simplify each side of the equation. Then combine like terms.

$$
\begin{aligned}
10 c-6(2 c-1) & =-2(c-3) \\
10 c-12 c+6 & =-2 c+6 \\
-2 c+6 & =-2 c+6
\end{aligned}
$$

Step 2 Add $2 c$ to both sides of the equation.

$$
\begin{aligned}
-2 c+6 & =-2 c+6 \\
-2 c+6+2 c & =-2 c+6+2 c \\
6 & =6
\end{aligned}
$$

Step 3 Interpret the resulting equation.

$$
6=6
$$

This is a true statement, so the original equation has infinitely many solutions.
Solution The equation $10 c-6(2 c-1)=-2(c-3)$ has infinitely many solutions.

## Example 5

Mr. Williams is buying tickets for the school musical. Adult tickets cost $\$ 3$ more than student tickets. He buys 7 student tickets and 6 adult tickets, and he spends a total of $\$ 83$. What is the price of one adult ticket?

## Strategy Write and solve an equation that models the situation.

Step 1 Identify the variable.
Let $x$ represent the price of a student ticket.
Since adult tickets cost $\$ 3$ more than student tickets, let $x+3$ represent the price of an adult ticket.

Step $2 \quad$ Write an equation to model the situation.
Since $x$ represents the price of a student ticket, the cost of 7 student tickets is $7 x$.
Since $x+3$ represents the price of an adult ticket, the cost of 6 adult tickets is $6(x+3)$.


The equation is $7 x+6(x+3)=83$.

Step 3 Solve the equation using the properties of equality and the distributive property, and by combining like terms.

$$
\begin{aligned}
7 x+6(x+3) & =83 \\
7 x+6 x+18 & =83 \\
13 x+18 & =83 \\
13 x+18-18 & =83-18 \\
13 x & =65 \\
\frac{13 x}{13} & =\frac{65}{13} \\
x & =5
\end{aligned}
$$

Step 4 Interpret your result.
Since $x$ represents the price of a student ticket, the price of a student ticket is $\$ 5$.
Since $x+3$ represents the price of an adult ticket, the price of an adult ticket is $\$ 8$.

## Solution The price of one adult ticket is $\$ 8$.

## COACHED EXAMPLE

Solve for $x .4(1+3 x)=5(x-2)$
Apply the distributive property to both sides of the equation.

$$
4(1+3 x)=5(x-2)
$$

$4+$ $\qquad$ $x=$ $\qquad$ $x$ - $\qquad$
Move the variable term from the right side of the equation to the left side by subtracting $\qquad$ from both sides. Then simplify each side.

$$
\begin{aligned}
4+12 x-\ldots & =5 x-10- \\
4+\ldots & =
\end{aligned}
$$

$\qquad$

Isolate the term containing the variable by subtracting $\qquad$ from both sides of the equation.
$4+$ $\qquad$ $x-$ $\qquad$ $=-10-$ $\qquad$
$\qquad$ $x=$ $\qquad$
Isolate the variable by dividing both sides of the equation by $\qquad$ —.
$\qquad$ $x \div$ $\qquad$ $=$ $\qquad$ $\div$ $\qquad$

$$
x=
$$

$\qquad$
Check that the solution makes the original equation true.

$$
4\left(1+3\left(\_\right)\right) \stackrel{?}{=} 5\left(\_-2\right)
$$

The solution to $4(1+3 x)=5(x-2)$ is $\qquad$ .

1 Compare the solution of each equation to 0 . Write the equation in the correct box.
$5(d+2)=3(d-6)$

$$
-4=\frac{1}{2} p-7
$$

$$
-6 m=2(3 m-1)
$$

$$
15-(4 z+3)=12
$$

$$
0.4(3.2 x+2)-x=2 x+1.8
$$

| Solution Is Less Than 0 | Solution Is Equal to 0 | Solution Is Greater Than 0 |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |

(2) Which equation has at least one solution? Circle all that apply.
A. $2 x-1=2$
B. $3(y+1)=3 y$
C. $5 p-(3+p)=6 p+1$
D. $\frac{4}{5} m=1-\frac{1}{5} m$
E. $10+0.5 w=\frac{1}{2} w-10$
F. $4 a+3(a-2)=8 a-(6+a)$
(3) Jamal is building a rectangular deck. The length of the deck will be 1 foot longer than twice the width. The deck will be attached to Jamal's house on one of its longer sides, and a railing will be attached to the other sides. Jamal calculates that he will need 49 feet of railing. What are the dimensions of the deck? Show your work.

4 Is each equation equivalent to $-4(3 x+2)=x+2(x-1)$ ? Select Yes or No.
A. $-12 x+2=3 x-1$
$\bigcirc$ YesNo
B. $3 x+2=x+2(x-1)+4$YesNo
C. $-12 x-8=2 x-2$
$\bigcirc$ Yes
O No
D. $-8=15 x-2$
$\bigcirc$ Yes $\bigcirc$ No
E. $-15 x=-10$
$\bigcirc$ Yes No
F. $x=-\frac{2}{5}$No

5 Select True or False for each statement.
A. If $\frac{4}{3} h=12$, then $h=16 . \quad$ True $\bigcirc$ False
B. If $-\frac{2}{5} r=-20$, then $r=50$.False
C. If $\frac{1}{2}=-\frac{3}{7} q$, then $q=-1 \frac{1}{6}$.TrueFalse
D. If $\frac{3}{8}=24 c$, then $c=9$.TrueFalse

6 Eight more than twice a number is equal to ten less than five times the number.

## Part A

Let $n$ represent the number. Write an equation that can be used to find $n$.
$\square$

## Part B

Solve your equation for $n$. Show your work.

7 The following equation is true for all values of $x$. Write the number that completes the equation. Show your work.

$$
5 y+2(3 y-1)=\quad y-(y+2)
$$

8 Explain why the equation $10 x-1=10 x+4$ has no solution.
$\square$

9 For each linear equation in the table, indicate with an " $X$ " whether the equation has no solution, one solution, or infinitely many solutions.

| Equation | No Solution | One Solution | Infinitely Many <br> Solutions |
| :---: | :---: | :---: | :---: |
| $8(a+2)=5 a+16+3 a$ |  |  |  |
| $6 m+2-4 m=2(m+2)$ |  |  |  |
| $3(z+3)=7+3 z+6-z$ |  |  |  |

10 Emma substitutes 3 for $x$ in a one-variable linear equation and finds that it makes the equation true. She then substitutes 5 for $x$ in the same linear equation and finds that 5 also makes the equation true. What can you conclude about the number of solutions of the equation? Explain your reasoning.

11 Amelia is making bags of snack mix for a class party. The snack mix includes dried fruit, cashews, and peanuts. The dried fruit costs $\$ 8.25$ per pound, the cashews cost $\$ 5.99$ per pound, and the peanuts cost $\$ 3.99$ per pound. Amelia buys 2 more pounds of peanuts than she does cashews and 1 pound of dried fruit. If her total bill is $\$ 41.18$, how many pounds of peanuts does she buy? Show your work.

12 Liam is solving the equation $12 a-4(5 a-1)=2(3 a+6)-4 a$. The result of each step of his solution is shown below.

$$
\begin{aligned}
12 a-4(5 a-1) & =2(3 a+6)-4 a \\
12 a-20 a+4 & =6 a+12-4 a \\
-8 a+4 & =2 a+12 \\
-6 a+4 & =12 \\
-6 a & =8 \\
a & =-\frac{4}{3}
\end{aligned}
$$

## Part A

Circle the step in which Liam's first error occurred. Describe the error.
$\square$

## Part B

Solve the equation correctly. Show your work.


